

# Stop this runaway truck, please

Road Runner and Coyote are up to their usual antics.

This time, Coyote decides to use his eighteen-wheeler to catch Road Runner. He remembers his most recent experience. He was travelling down a mountain road at a tremendously fast speed, when his brakes suddenly failed.



His only recourse was to crash into the side of the mountain to stop the truck. But this time, Coyote has a plan. He will build an escape ramp, so that if his brakes fail, his runaway truck will come to a safe stop. But he has a problem: the amount of space is limited.

What should be the grade (slope) of Coyote's escape ramp?

## Forensics objectives:

- Students will be able to be curious about the uses of mathematics in real life.

## Physics and Mathematics objectives:

- Students should be able to use the Pythagorean Theorem, compute slope, and plot coordinates.
- Students should understand Galileo's theory of motion.

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- Students will be able to gather data for correlations between height, horizontal displacement, and road surface.
- Students will be able to determine which regression model of data to predict height based on horizontal displacement.

### **MONE objective:**

- Gerçek hayat durumlarıyla ilgili bir istatistik problemini çözmek için verileri toplar, düzenler, temsil eder ve yorumlar.

### **Materials:**

Each group of students needs the following materials:

- TI-84 Plus Family
- Two boards (about 3 feet long and 1 foot or more wide)
- Full noncarbonated beverage can, a physics car, or a heavy, free-rolling toy car.
- Metersticks and rulers
- Books, magazines, and cardboard to prop up boards for ramps.
- Masking tape
- Copy of experiment directions
- Activity sheets for each students

### **Procedure:**

This worksheet is assigned as group work. So, each group of students set up two ramps represents a mountain incline; the other represents a truck escape ramp. The vehicle, that is, the can or car, will roll down the mountain ramp and up the escape ramp.

Use cardboard, as needed, to eliminate the bumps between the end of each board and the floor. Run a strip of masking tape the entire length of the truck escape ramp. Beginning of the bottom of the ramp, mark the tape in convenient units of length, such as half-inches or centimeters. Separate the two ramps by a distance of from at least one can or car lengths to as much as three feet. Adjust the height of two ramps so that the can, car, or ball builds enough

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momentum to roll to the top of the truck escape ramp without rolling off. The height of the truck escape ramp is now the minimum height. The grade, or slope, of the escape ramp increases during the experiment; initially, the grade should be as small as possible.

In this experiment, you will roll a can, car, or ball down a ramp representing the mountain road and record the stopping distance, that is, the distance that it travels up the escape ramp. You will run this experiment several times. Each time, you will increase the height of the escape ramp by a small increment.

1) From the problem statement, explain the purpose of the experiment.

2) Before you collect your data, make a conjecture about this experiment. If you are considering the stopping distance versus the grade of the escape ramp, what will the graph of your data look like? Predict the grade of Coyote's escape ramp. Are stopping distance and height of mountain are independent? Explain your reasoning.

3) The grade of a road is

$$\frac{\text{height}}{\text{horizontal displacement}} \times 100$$

As shown in the sketch. The grade is usually expressed as a percent.

Consider for example, a five mile long, or 26,400 foot-long, section of highway that rises (or drops) 1850 feet. What is the grade of this stretch of highway? Use Pythagorean Theorem to find the horizontal displacement.

$$26400^2 = 1850^2 + (\text{horizontal displacement})^2$$

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Therefore, the horizontal displacement is 26,335 feet, rounded to the nearest integer.

The grade is,

$$\frac{1850}{26335} \times 100$$

or about 7 percent.

- How can you find the horizontal displacement of the ramp when its height and length are known?

- What two pieces of information do you need to compute the grade of the ramp?

Measure the following lengths, and record them in table. Each group selects one roller, one of two spotters, and a recorder.

- Length of the mountain road
- Height of the mountain road
- Length of the escape ramp
- Height of the escape ramp

Setup	Length	Height	Horizontal displacement	Grade
Mountain road				
Escape ramp				

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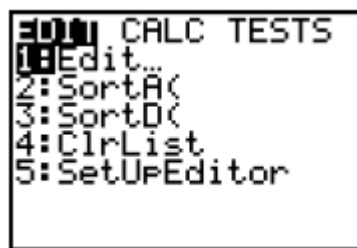
Hold the can, car, or at the top of the mountain road. Release it. Observe the distance that it travels up the escape ramp before it rolls backward. Record this value. Repeat the procedure two more times, and find the mean or median of the three distances.

Increase the ramp height by a small increment. Run the experiment again. Repeat this process to obtain at least six sets of data.

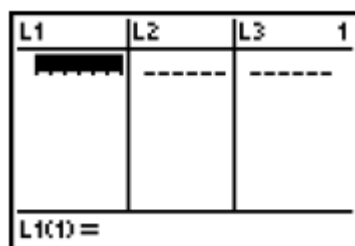
Record the height of the escape ramp and the stopping distances in table below. Find the mean or median stopping distance of the three trials.

Escape ramp height	Horizontal displacement	Grade	Trial 1	Trial 2	Trial 3	Trial 3	Mean or median

Press **STAT**; 1: Edit ... should be highlighted. Press **ENTER**.



Use the cursor keys to highlight L1. Press **CLEAR**; then press **ENTER**. List L1 should be cleared of all data. Repeat the process for lists L2 and L4.



Use the left cursor key to h **L1(1) =** Enter the values from your height columns. Press **ENTER** after each number.

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Use the right cursor key to move to labeled L2. Use the up cursor key to highlight L2. Type in the expression,  $\sqrt{(\text{lenght of ramp})^2 - L1^2}$

Press **ENTER**. The values for the horizontal displacement should appear in this list. Record these values in table.

Use the right cursor key to move to the column labeled L3. Use the up cursor key to highlight L3. Type in expression  $(L1/L2) \times 100$ . Press **ENTER**. The grade for each corresponding height should appear in the column. Record these values in your table.

Use the right cursor key to move to column labeled L4. Enter the mean or median stopping distance your table.

- Make a scatter plot of the data.

To view graph of your data on the calculator, press **2ND Y=**. Press **ENTER** to select to plot 1.



Press **ENTER** to turn on the plot. Press the down cursor key. The first icon (the scatterplot icon) will flash. Press **ENTER** to select the scatterplot icon. Press the down cursor key. Next to Xlist, enter L3 by pressing **2ND 4**. Use the down cursor key to highlight the box. Press **ENTER**.



Press **ZOOM**. Use the down cursor key to select 9: **ZoomStat**. Press **ENTER**.

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4) What type of function might the curve represent?

- How does this graph compare with the graph that you drew?
- Describe what you see on your graph? Are the linear?
- What do your data and the graph tell you about physical situation?
- Does connecting your points with a smooth curve make sense? Explain.
- Does extending the curve beyond your data points make sense? Explain.
- Draw a smooth curve through the data points. State the new domain.
- Does an x-intercept occur?
- If it does, explain its relationship to the physical situation. If it does not, explain why not.
- Does a y-intercept occur?
- If it does, explain its relationship to the physical situation. If it does not, explain why not.

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- 5) Select the grade between your minimum and maximum escape-ramp grades. Use your curve to predict the corresponding stopping distance:

Grade:

Stopping distance:

- 6) Use the half the escape-ramp length as your stopping distance. Use your curve to predict the corresponding grade.

Stopping distance:

Distance:

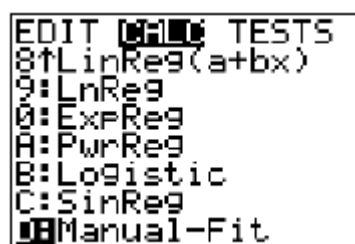
- 7) Coyote will succeed for the first time because of your input. What advice can you give him?
- 8) Consider all the families of curves with which you are familiar. Choose a family of curves, and try to find an equation that closely first your data. What family of curves did you choose?

- Determine the best-fit regression equation for this graph.

Press **STAT**. Use the right cursor key to highlight **CALC**. Use the down cursor key to select a regression model (option 5,6,7,8,9,0,A,B,or C). Press **ENTER**. The regression model that you selected appears on the home screen. Type L3,L4,Y1.( To type L3, you must press **2ND** 3; to type L4, press **2ND** 4;and to type Y1, press **VARS**. Use the right cursor key to highlight **Y-VARS**. Press **ENTER** to select 1:Y1. Repeat for each type of equation. Which model appears to provide the best fit?



Press **STAT** to display the Stat menu. Select **CALC**. Press several times to scroll down to select D:**Manual-Fit**. Press **ENTER**. This displays a free-floating cursor at the center of the display screen.





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- Are you satisfied with your equation? Do you think that you could have found a better-fitting model? Explain.
- How does your graph change if you use height as the independent variable?
- How does your graph change if you use angle measure as the independent variable?

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## Calculating chi-squared

The chi-square goodness-of-fit test is used to determine whether an observed frequency distribution is significantly different from the expected distribution, or how “good” (sic) the two distributions fit each other.

Notice that: Your collected data are continuous. For calculating chi squared, you will have discrete data. According to following interval, you will complete the tables.

Escape ramp height(x)	interval	frequency	Horizontal displacement(y)	interval	frequency
	$0 < x < 10$			$0 < x < 10$	
	$10 < x < 30$			$10 < x < 30$	
	$30 < x < 50$			$30 < x < 50$	
	$50 < x < 60$			$50 < x < 60$	
	$60 < x < 70$			$60 < x < 70$	
	$70 < x < 90$			$70 < x < 90$	

You determined a question which is “Are stopping distance and height of mountain are independent?”. You made a table and you stated  $H_0$  called null hypothesis. In this stage, you will built-in chi-square goodness-of-fit (GOF) test.

- State the alternative hypothesis called  $H_1$ .
- Enter the observed data above. (The general procedure is to put observed frequencies in L1)
- Now you need to compute the expected frequency about height of mountain and stopping distance. Hint: you calculate the expected frequency as  $E = n/k$ , where  $n$  is total number of trials of stopping distance, and  $k$  is the number of different height of mountain. Enter the expected frequency in L2.

L1	L2	L3	1
████████	-----	-----	
L1() =			

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The next step is to compute  $\frac{(O-E)^2}{E}$  for each category. You will do that by highlighting L3 and then typing the following keystrokes: (2ND 1-2ND 2) $\chi^2 \div$ 2ND 2 **ENTER**. These keystrokes will enter the formula shown in the calculator display. If you have entered the formula correctly, L3 will fill up with the values shown in the display to the right.

Press **2ND MODE** to quit the stat editor and return to the home screen. Once you add up all of the entries in L3, you will have found the value of your chi-squared test statistics:

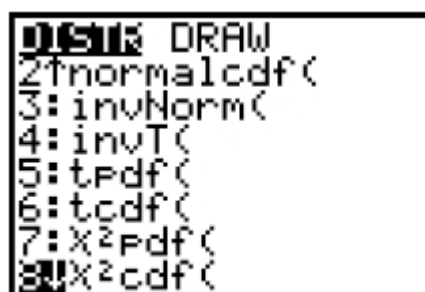
$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

You will use the sum function to do so. Press **2ND STAT** and scroll right to the MATH menu. Then, scroll down to 5: sum (and press **ENTER**. Type **2ND 3**) to indicate that you want to sum the contents of L3 and then press **ENTER**.



Next, we need to find the p-value associated with your chi-squared test statistics. To do so, press **2ND VARS** to get **DISTR** menu. Scroll down to 8:  $\chi^2$  cdf

And press **ENTER**. You will find the probability of obtaining chi-squared test.



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- Find the number of degrees of freedom by subtracting one from the number of categories.  
 $df = (r-1)(c-1)$  for a contingency table which  $r \times c$  in size.
- Round chi-squared values to 3 decimal places and p-values to 3 significant figures.
- What are your p-values? What does this mean?
- Determine significant level.

You will state rejection inequality

$$\chi_{calc}^2 > k$$

where k is obtained from the table of critical values.

- From the contingency table, find chi-squared using formula.
- Determine whether accept  $H_0$  or reject  $H_0$ , depending on the rejection inequality result. Hint: if operating at a 5% level, you could also use p-values to help you with your decision making.

If  $p > 0.05$ , we accept  $H_0$ .

If  $p < 0.05$ , we reject  $H_0$ .

Moreover,

It is relatively easier to conduct chi-squared goodness-of-fit test.

Once you have entered the observed frequencies in L1 and expected frequencies in L2, press **STAT** and scroll right to the **TESTS** menu. Then scroll down to: D and press **ENTER**. Your calculator will prompt you for the list with the observed frequencies (L1 for example)

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the list with between the expected frequencies (L2 for this example), and the number of degrees of freedom. Next, highlight Calculate and press **ENTER**. It gives you both value of the chi-squared test statistic and p-value.

This worksheet exercise that you will conduct during class tonight will provide you with an example of how to conduct chi-square goodness-of fit-test in situations where the expected frequencies are not equal.

### Investigation chi-squared in excel

You can use excel for chi-squared test

with your discrete data.

- 1) Enter your collected discrete data in row and column.
- 2) Find sum of your data.
- 3) Calculate total values each column and row.
- 4) Find your expected value
- 5) Use CHITEST function of excel to find your Chi-square result.

Expected value

sum product  
124

	A	B	C	D	E	F	G
1		Male	Female	Total			
2	Not at all	25	25	50	0.403226		
3	Somewhat	18	19	35	0.282258		
4	Very	24	15	39	0.314516		
5	Total	65	59	124			
6							
7		26.20968	23.79032				
8		18.34677	16.65323				
9		20.44355	18.55645				
10							
11							

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## **Reflection**

In this assignment, I prepared worksheet which includes physics, mathematics and technology for students. The integration reminds students to be importance of STEM education. As MONE's official approach to technology in mathematics teaching, the technology benefits students in mathematics classroom in terms of their problem solving skills, reasoning. The technology also develops teaching and learning environment in mathematics classroom. Moreover, the technology has important roles for synthesis, discussing in teaching and learning process (Koca, 2009). In this worksheet, students feel the positive effects of technology by using TI-calculator.

## **References**

National Council of Teachers of Mathematics (2011). When 95% accurate isn't: Exploring Bayer's Theorem. *Mathematics Teacher*, 104(6).

Aslı Özgün-Koca (2009). The Views of preservice Teachers About the Strengths and Limitations of the Use of Graphing Calculators in Mathematics instruction. *Jl. of Technology and Teacher Education* 17 (2), 203-227.