

ALGEBRA WORKSHEET FOR 10TH GRADERS

SUBJECT: COMPLEX NUMBERS

NECESSARY MATERIALS: TI-84 CALCULATOR, EXCEL

NECESSARY APPLICATIONS: POLYNOMIAL ROOT FINDER AND SIMULTANEOUS EQUATION SOLVER.

Download link:

<http://education.ti.com/en/us/software/details/en/7DFF09A5B117420D8BE99109F1B36D34/83polynomialrootfinderandsimultaneousquationsolver>

For the Excel based questions you need to install an Add-In called Analysis ToolPak. To do this click on the office button and click excel options. Go to Add-Ins on the left hand side and add Analysis ToolPak (Çözümleme Araç Takımı). Now you can express complex numbers in Excel.

KAZANIMLAR:

10.6.1.2. $i = \sqrt{-1}$ sanal birim olmak üzere bir karmaşık sayının $a + bi$ ($a, b \in \mathbb{R}$) biçiminde ifade edildiğini açıklar.

1. Diskriminantın sıfırdan küçük olduğu durumlarda ikinci dereceden bir denklemin köklerinin bulunabilmesi için gerçek sayılar kümesini de kapsayan yeni bir sayı kümesi tanımlama gereği örneklerle açıklanır.

2. $i = \sqrt{-1}$ şeklinde tanımlandığı belirtilir. i sayısının kuvvetleri hesaplandırılır.

3. Karmaşık sayılarda toplama, çarpma ve bölme işlemleri ve özellikleri gösterilir.

4. Bir karmaşık sayının eşleniği, modülü ve argümanı verilir.

We can't do everything on \mathbb{R} !

Turn on your TI-84 calculator. Press \square key and choose PlySmlt2 application.

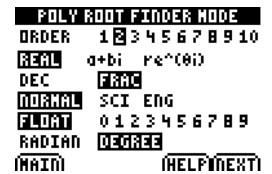
Press any key. Choose the first item poly root finder (or just hit 1) from the main menu



+

Don't make any changes. Press σ to go next. Now, we have a polynomial of second degree.

1) Enter the following coefficients. After you enter the numbers press the σ key to solve and after the application finds the roots press the θ key twice to go back and clear. Write the roots down.



a) $a_2 = 1, a_1 = 0, a_0 = -16$

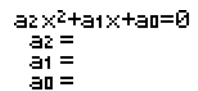
$x_1 = \quad, x_2 = \quad$

b) $a_2 = 1, a_1 = 2, a_0 = 1$

$x_1 = \quad, x_2 = \quad$

c) $a_2 = 1, a_1 = 0, a_0 = 4$

$x_1 = \quad, x_2 = \quad$



What happened after you entered the coefficients in c)?

What does no real roots found mean?



Let's go back to the mode menu by pressing the π key.

Move the cursor down and choose a+bi. Go next and try again.

Now the application can find the roots no problem.

What did you notice about the roots? Why aren't they real numbers?

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POLY ROOT FINDER MODE
ORDER 1 2 3 4 5 6 7 8 9 10
REAL a+bi re^iθ(i)
DEC     FRAC
NORMAL SCI ENG
FLOAT  0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
(MAIN) (HELP/NEXT)

```

What is an imaginary number?

Go back to the home screen. Press the ζ key. Choose a+bi format just like you did with the previous application.

2) For square root press $\sqrt{\quad}$ keys. Calculate:

- $\sqrt{-1} =$
- $\sqrt{-9} =$
- $\sqrt{-25} =$
- $\sqrt{-7} =$
- $\sqrt{-16} =$
- $\sqrt{-8} =$

Now that we've established $i = \sqrt{-1}$,

3) To take a power of some number press \square key and for i press $\sqrt{-1}$. Calculate

$$i^2 = \qquad i^3 =$$

$$i^4 = \qquad i^5 =$$

$$i^6 = \qquad i^7 =$$

$$i^8 = \qquad i^9 =$$

$$i^{10} = \qquad i^{11} =$$

$$i^{12} = \qquad i^{13} =$$

What do you notice about the powers of i?

Bonus question: Can you create a formula to find the value of i^{158} without using the calculator?

Complex Numbers and their algebraic properties

A complex number is of the form $a+bi$. Complex numbers and the imaginary number are not the same things!

Find the real and imaginary parts of the complex numbers below. Press \square choose complex (CPX) and for the real part choose real(or press 2) and for the imaginary part choose imag (or press 3).

4)

a) $z = 5+7i$ $\text{Re}(z) =$ $\text{Im}(z) =$

b) $z = 9-4i$ $\text{Re}(z) =$ $\text{Im}(z) =$

c) $z = \sqrt{5}+(\sqrt{3}i/2)$ $\text{Re}(z) =$ $\text{Im}(z) =$

d) $z = 16-3i$ $\text{Re}(z) =$ $\text{Im}(z) =$

Addition and Subtraction

5) Calculate:

a) $(17+8i) + (-2+3i) =$

b) $(12-9i) + (-10 - 6i) =$

c) $(-88+11i) - (78+15i) =$

d) $(-24-6i) - (-28+6i) =$

e) $(18-15i) + (4+9i) - (11+20i) =$

f) $(15+7i) - (-11-9i) + (5+3i) =$

Multiplication

6) Calculate:

a) $-20i*(3+6i) =$

b) $(-15-9i) * (-8-5i) =$

c) $-i * (3+4i) * (7i-6) =$

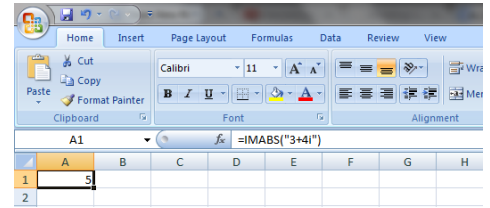
d) $(1+i) * (1-i) =$

e) $(7-8i) * (7+8i) =$

f) $(\sqrt{2}-6i) * (\sqrt{2}+6i) =$

What do you notice about the answers in d), e) and f) ? Why don't they contain i terms?

Bonus question: Show without using the calculator, show $\text{Re}(iz) = -\text{Im}(z)$ and $\text{Im}(iz) = \text{Re}(z)$, where $z = a+bi$.



Complex Conjugation

7) To find the conjugate press choose complex (CPX) and choose conj (or press 1)

a) $5+i$

b) $9 - 3i$

c) $10 + 5i$

d) $3 - \sqrt{2}i$

Division

8) Calculate

a) $1/i =$

b) $(4-i)/3i =$

c) $(10-3i)/(4+6i) =$

d) $(i+7)/(7i+15) =$

Bonus question: Without using the calculator, calculate $-3/(3-i)$ by multiplying the numerator and the denominator by the conjugate of $(3+i)$. What do you notice about the result?

We can work with complex numbers on Excel too!

To find the absolute value (or modulus) of a complex number we need the IMABS function.

Enter `=IMABS("z")` to the function area where $z=a+bi$

9) Calculate the moduli of the below complex numbers

a) $3+4i$

b) $5+12i$

c) $1+3i$

d) $-9+i$

To find the angle (or argument) of a complex number we need the IMARGUMENT function.

Enter =IMARGUMENT("z") to the function area where $z=a+bi$.

10) Calculate the arguments of the below complex numbers

a) $2+3i$

b) $10-6i$

c) $1+\sqrt{3}i$

d) $\sqrt{2}-i$

(IMARGUMENT function calculates the radian angle, so you can find the DEGREES() function (to convert) useful here.)

