

WORKSHEET FOR 12 GRADERS
CALCULUS

Subject: Learning more about 0^0 .

Necessary Equipments: Ti-84 Plus calculator, computer, pencil, note book.

Necessary applications: You need to write a small program.

Time: Worksheet should be completed in one and half hour.

Notes: While you are using your TI-84 Plus Calculator do not forget minus - and negative signs (-) are different. If you press minus instead of negative sign you will get automatically syntax error.

This is an individual weekend worksheet and students do not allowed to ask help from their parents.

Kazanımlar:

İD.12.1.2.2. Bir fonksiyonun bir noktada ve bir aralıkta türevli olmasını inceler.

[R] Tanım kümesi açıkça belirtilmemiş bir fonksiyonun tanım kümesi olarak, fonksiyonun kuralın geçerli olduğu en geniş küme alınır.

[R] Fonksiyonun türevli olmadığı noktalarla grafiği arasında ilişki kurulur.

[R] Bilgi ve iletişim teknolojilerinden yararlanır.

Please read the pages about investigation before you start to do worksheet. They will be very helpful for you to look from different perspectives to the topic.

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An Investigation: What does 0^0 equal?

Student 1:

I know!

$$x^0 = x^{1-1} = x^1 x^{-1} = \frac{x}{x} = 1.$$

Now we just plug in $x=0$, and we see that zero to the zero is one!

Student 2:

No, you're wrong! You're not allowed to divide by zero, which you did in the last step. This is how to do it:

$$0^x = 0^{1+x-1} = 0^1 \times 0^{x-1} = 0 \times 0^{x-1} = 0$$

which is true since anything times 0 is 0. That means that

$$0^0 = 0.$$

Student 3:

That doesn't work either, because if $x = 0$ then

$$0^{x-1} \text{ is } 0^{-1} = \frac{1}{0}$$

so your third step also involves dividing by zero which isn't allowed! Instead, we can think about the function x^x and see what happens as $x > 0$ gets small. We have:

$$\lim_{x \rightarrow 0^+} x^x = \lim_{x \rightarrow 0^+} \exp(\log(x^x))$$

$$= \lim_{x \rightarrow 0^+} \exp(x \log(x))$$

$$= \exp(\lim_{x \rightarrow 0^+} x \log(x))$$

$$= \exp(\lim_{x \rightarrow 0^+} \frac{\log(x)}{x^{-1}})$$

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$$= \exp\left(\lim_{x \rightarrow 0^+} \frac{\frac{d}{dx} \log(x)}{\frac{d}{dx} x^{-1}}\right)$$

$$= \exp\left(\lim_{x \rightarrow 0^+} \frac{x^{-1}}{-x^{-2}}\right)$$

$$= \exp(\lim_{x \rightarrow 0^+} -x)$$

$$= \exp(0)$$

$$= 1$$

So, since $\lim_{x \rightarrow 0^+} x^x = 1$, that means that $0^0 = 1$.

High School Teacher:

Showing that x^x approaches 1 as the positive value x gets arbitrarily close to zero does not prove that $0^0 = 1$. The variable x having a value close to zero is different than it having a value of exactly zero. It turns out that 0^0 is undefined. 0^0 does not have a value.

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Questions

1) Before you begin to solve this worksheet do small interviews with your parents and a few friends to learn their ideas about the answer of 0^0 . Write a small journal that reflects your findings and own ideas about the question.

2) Complete the table below by using your graphing calculator. Write your answers in column C. After completing the column C what can you say about the value of 0^0 ? How did you decide? Try to calculate the value of 0^0 by using your calculator. What did you find? Why do you think it is like that? Explain.

A	B	C
$0.1^{0.1}$		
$0.01^{0.01}$		
$0.001^{0.001}$		
$0.0001^{0.0001}$		
$0.00001^{0.00001}$		
$0.000001^{0.000001}$		
$0.0000001^{0.0000001}$		

To calculate the values you should follow the steps below:

- Turn on your calculator and write the value (for example: write 0.1)

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- Dot notion stays at the bottom between the [0] and [(-)] button.
- To calculate $0.1^{0.1}$ you should write $0.1^0.1$ by using [^] button.
- You should press [ENTER] button to see the answer.
- Follow the same steps for each calculation.

Your basic steps should be like:

```
0.1^0.1■          0.1^0.1
                   .7943282347
                   ■
```

- 3) Graph $y=x^x$ between the intervals [0, 1] and [0, 1]. When $x=0$ what it seems the value of y ? Activate trace tool to study the graph near and $x=0$ and compare your result with the table above (see question 2). What did you find out? Explain.

To graph the equation follow the steps below:

- Press [Y=] button.

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- Write the equation $Y_1=X^X$.
- Press [WINDOW] button.
- Write $X_{min}=0$, $X_{max}=1$, $Y_{min}=0$, $Y_{max}=1$.
- Press [GRAPH] button.
- Press [TRACE] button and begin to move the cursor to see different X and Y values.

Your basic steps should be like:

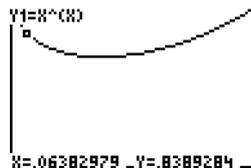
```

Plot1 Plot2 Plot3
Y1=X^X
-----
Y2=
Y3=
Y4=
Y5=
  
```



```

WINDOW
Xmin=0
Xmax=1
Xscl=1
Ymin=0
Ymax=1
Yscl=1
Xres=1
  
```



- 4) By using your calculator find the approximate value of x between 0 and 1 for which the value of y is the smallest. What is the minimum value of y? How did you decide? After that find the exact value of x for which y is as small as possible by using pencil and paper. Compare your results.

To find the approximate value of x follow the steps below:

- After graphing $Y=X^X$ press [2ND] and [TRACE] button.

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- Press 3 or scroll all the way down and highlight minimum. After that press [ENTER] button.
- Write for a value for left bound (for example 0) press [ENTER].
- Write for a value for right bound (for example 1) press [ENTER].
- Write for a value for guess (for example 0.5) press [ENTER].
- You will see for which x value y is the smallest.

Note: Different results can be found if other choices are made for right bound, left bound and guess.

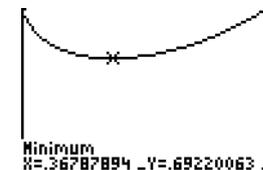
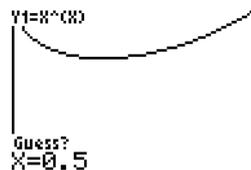
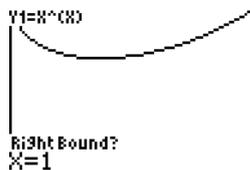
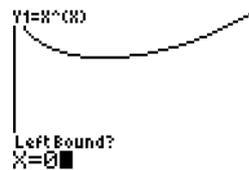
Your basic steps should be:

```

Plot1 Plot2 Plot3
┌───┴───┬───┴───┬───┴───┐
√Y1= X^X
└───┬───┬───┬───┘
√Y2=
√Y3=
√Y4=
√Y5=
    
```

```

CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
    
```



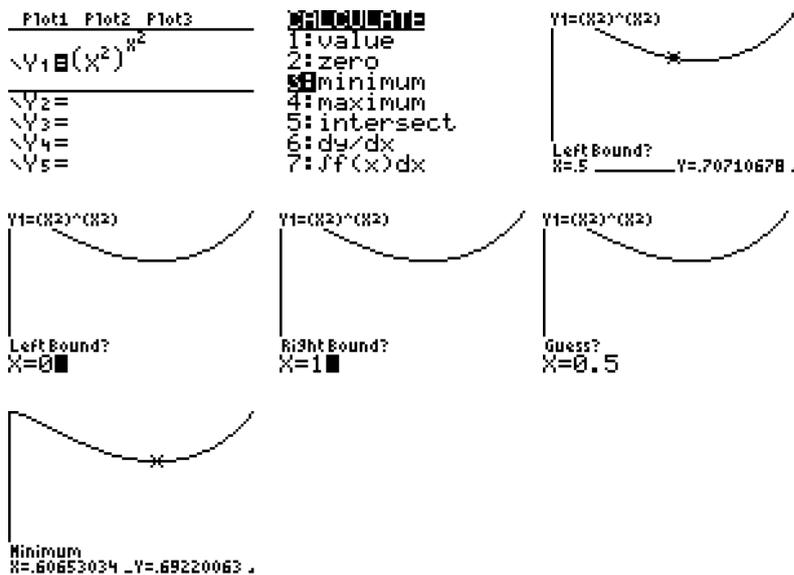
- 5) Let $y=(x^r)(x^r)$ for each of the values of r in the table below. Find the value of x between the intervals 0 and 1 for which y is as small as possible and find the smallest value of y.

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A	B	C
r	a (value of x for which the function is the smallest)	b (smallest value of y)
1		
2		
3		
4		
5		

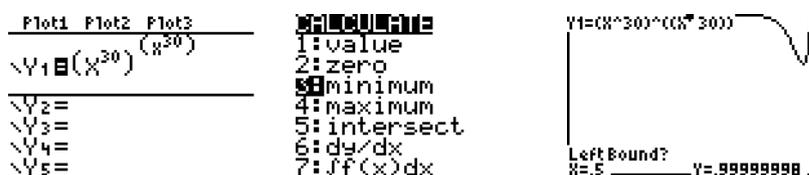
To calculate the values follow the same steps in the fourth question. Do not forget to change your equation according to r value.

For example if you take $r=2$, your basic steps should be like:

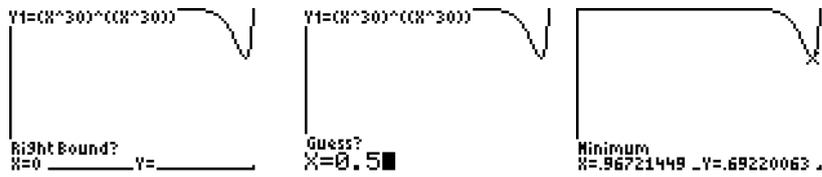


6) Look at your findings in the previous question. Suggest a rule for a and b in terms of b. Use this rule to predict the values of a and b for $y = (x^{30})^{(x^{30})}$. Graph this function. Then calculate the values of a and b. Compare solutions with your prediction. Are they similar or different from each other? If so why? Why do you think like that?

Your basic steps should be like:



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- 6) Now find the exact values of a and b in terms of r for which y is as small as possible by using pencil and notebook. Are these results consistent with your findings in question 5? Explain your reasoning.

- 7) I saw the house below in a very rich neighborhood of Ankara. It was something special with that house for me because on the fences of this house it was written number 0^0 . Then I took a photo of this beautiful house and paste into Geometer's Sketchpad and positioned so that the point $(0,0)$ coincided with the lower left corner of the top section of the window. Five points A, B, C, D and E were placed along the top

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curve of the window (see Figure 2) and then used to form four trapezoids. The coordinates of these points were measured and recorded in the table below.



Figure 1



Figure 2

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	A	B
1	A	(0, 4.00)
2	B	(-1.06, 3.44)
3	C	(-2.30, 2.99)
4	D	(-3.62, 2.49)
5	E	(-4.97, 2.01)

a) Find the combined area of the four trapezoids. How is this area related to the area of the curved section of the window as it appears in Figure 1?

To find the area of one trapezoid you should write a small program to your calculator:

- Press [PRGM] button.
- Highlight NEW by using right arrow. Press [ENTER] button.
- Give a name to your program. To use the letters first press [ALPHA] button then letter.
- Press [ENTER] button.
- Press [PRGM] button. Highlight I/O and select Prompt.
- Press [ENTER] button.
- Write "A, B". Press [ENTER].
- Write A+B then Press [STO] button and write C. (it basically says that A+B equals C)
- Press [ENTER] button.
- Write the formula there $(0.5)*C$ press [STO] button and write D.
- Press [PRGM] and highlight I/O and select Disp. Press [ENTER]. Write "D" there.
- Write [2ND] and [MODE] to quit your program.

Note: Do not forget that with the help of this program you can find trapezoid's area separately. If you want to find the total area of the shape you should add them up.

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Your basic steps should be:

<pre>EXEC EDIT [F2] [F3] Create New</pre>	<pre>PROGRAM Name=AREA</pre>	<pre>CTL [F10] EXEC 1: Input 2: Prompt 3: Disp 4: DispGraph 5: DispTable 6: Output(7: getKey</pre>
<pre>PROGRAM: AREA : Prompt A,B</pre>	<pre>PROGRAM: AREA : Prompt A,B : A+B>C</pre>	<pre>PROGRAM: AREA : Prompt A,B : A+B>C : (.5)*C</pre>
<pre>CTL [F10] EXEC 1: Input 2: Prompt 3: Disp 4: DispGraph 5: DispTable 6: Output(7: getKey</pre>	<pre>PROGRAM: AREA : Prompt A,B : A+B>C : (.5)*C+D : Disp D</pre>	<pre>Pr9mAREA A=?1 B=?1.29 1.145 Done</pre>

Did you notice anything with the formula above? You know the formula of trapezoids area. What is missing there? Why do you think it is like that? Explain your reasoning.

- b) Find the total area without using calculator and compare your solutions. What did you notice about the solutions? Are they similar or different from each other? Why?

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c) To find the actual area of the curved section of the window what else do you need to know about the photograph? List your answers and be ready for the class discussion.

Reflection on My Second Worksheet

In this brief assignment I will analyze my preparation process in order to prepare this worksheet above. My assignment will highlight some important techniques that I use for preparing worksheet and some difficulties that I faced during this process.

In order to prepare this worksheet with the help of our educator I decided to choose a topic which my students can use calculus in it. I choose the topic discovering the value of 0^0 because it was a very interesting topic for me. I wanted my students to use their calculators mostly while they were working on this worksheet. That is why I asked many questions that students can solve with the help of their calculators. I also asked some questions which students were not allowed to use their calculators in order to encourage them to use their calculus knowledge. Thus I had a chance to apply MONE objectives in my worksheet because students will use derivative and they also use technology at the same time. I again used programming because I wanted my students to be familiar with the programming. Also I

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wanted my students to see advantages of using programming. To gather students' attention I integrated the current topic with real life. To realize this aim I used a photograph and GSP program for mathematical inquiry. I hope it will encourage my students to see patterns and relationships with real life and mathematics by having fun.

This worksheet is a good example of formative assessment. Teachers can easily understand what students can do or cannot do by looking the results. Also interviewing and journal writing techniques used to encourage students to find further information about the topic and write down their findings. Moreover these techniques in other words alternative strategies will enable teachers to see students' working process. Preparing this worksheet made me think about different ideas and question types to motivate my students. Every time I prepare a worksheet I feel myself more confident and closer to be a real teacher. Nowadays nothing can make me happier than this feeling.

References

- No Author. (2010). *Q: What does 0^0 (zero raised to the zeroth power) equal? Why do mathematicians and high school teachers disagree?* Retrieved from <http://www.askamathematician.com/2010/12/q-what-does-00-zero-raised-to-the-zeroth-power-equal-why-do-mathematicians-and-high-school-teachers-disagree/>
- National Council of Teachers of Mathematics. (2010). Free throws, probability, and the golden ratio. *Mathematics Teacher*, 103(1), 478-481.