CALCULUS WORKSHEET FOR THE 12^{TH} GRADERS

LIMITS

<u>NECESSARY MATERIALS</u>: TI-84 CALCULATOR

<u>KAZANIMLAR:</u>

<u>İD.12.1.1.1</u>. Bir fonksiyonun bir noktadaki limiti, soldan limiti ve sağdan limiti kavramlarını tablo ve grafik kullanarak örneklerle açıklar.

- Limit kavramı bir bağımsız değişkenin verilen bir sayıya yaklaşmasından yola çıkılarak açıklanır.

- Limit ile ilgili özellikleri belirtir ve uygulamalar yapar.
- Bilgi ve iletişim teknolojilerinden yararlanarak fonksiyonların tablo ve grafik gösterimleri yardımıyla limit uygulamaları yaptırılır.

Notation and Definition

A limit is a value that a function approaches as the input approaches to some value. We denote limit as below.

$\lim_{x \to a} f(x)$=L reads: “The limit of f of x, as x approaches to a, is L.”

Augustin-Louis Cauchy and Karl Weierstrass formalized the definition of limit, as known as, a calculus class will be the **ε-δ** definition and you will hear a lot more from Cauchy and Weierstrass.

History

Augustin-Louis Cauchy

"More concepts and theorems have been named for Cauchy than for any other mathematician (in elasticity alone there are sixteen concepts and theorems named for Cauchy). Cauchy was a prolific writer; he wrote approximately eight hundred research articles and five complete textbooks. He was a devout Roman Catholic, strict Bourbon royalist, and a close associate of the Jesuit order.”

Karl Theodor Wilhelm Weierstrass

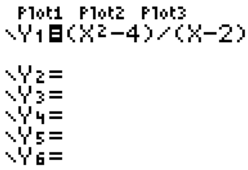
Weierstrass was interested in the soundness of calculus. At the time, there were somewhat ambiguous definitions regarding the foundations of calculus, and hence important theorems could not be proven with sufficient rigour. While Bolzano had developed a reasonably rigorous definition of a limit as early as 1817 (and possibly even earlier) his work remained unknown to most of the mathematical community until years later, and many had only vague definitions of limits and continuity of functions.

One Sided Limits

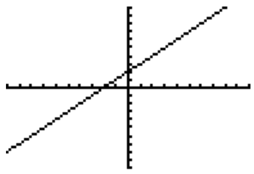
Consider the functions

f(x) = $(x^2-4)/(x-2)$ and g(x) = $(x^2-5)/(x-2)$.

Turn on your calculator. Press o buton and enter the function f(x) and then hit the graph button.

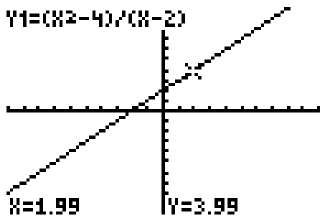


Your graph should look like this. You should notice that we can't say anything about this function for x=2. Let's have a little experiment then.



Now press ψ and ρ button and choose value (or just press 1). Now you have values for f(x) for every x you enter. This is a sample exercise (meaning it's a freebie for you.)

X	f(x)
1.9	**3.9**
1.99	**3.99**
1.999	**3.999**
1.9999	**3.9999**



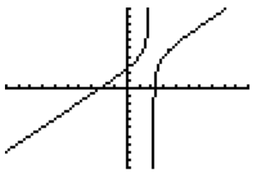
Notice as we move down in the table the x value gets closer to 2. We denote this by: $\lim_{x \to 2-} f(x)$ and we read as x approaches to 2 from the left. This is a one sided limit and f(x) or y values get closer to 4. As you can see $\lim_{x \to 2-} f(x)$=4.

1) Do the same thing for $\lim_{x \to 2+} f(x)$ (as x approaches to 2 from the right). Write down the **y** values.To what number the y values approach? In other words, $\lim_{x \to 2+} f(x)$=?

Are $\lim_{x \to 2-} f(x)$ and $\lim_{x \to 2+} f(x)$ equal to eachother? If so, what do you think that means? Any guesses?

Now we will do the exact same thing for g(x) = $(x^2-5)/(x-2)$.

Your graph should look like this. (It looks uncanny right?)



2) Find $\lim_{x \to 2+} g(x)$ and $\lim_{x \to 2-} g(x)$

X	**g(x)**
2.1	
2.01	
2.001	
2.0001	

x	**g(x)**
1.9	
1.99	
1.999	
1.9999	

Observe for x<2 g(x) increases without a bound. And for x>2 g(x) decreases without a bound too. Since there are no numbers g(x) approaches to we can say:

$\lim_{x \to 2+} g(x)$ and $\lim_{x \to 2-} g(x)$ do not exist.

So we can conclude , a limit exists if and only if both corresponding one-sided limits exist and are equal.

That is,

$\lim_{x \to a} f(x)$=L, for some number L, if and only if $\lim_{x \to a} - f(x)$=L and $\lim_{x \to a} + f(x)$=L

Limits

3) Decide if the limit, $\lim_{x\to 0} sinx/x$ exists by graphing f(x). Where f(x)=sinx/x

X	f(x)

x	f(x)

4) Decide if the limit $\lim_{x\to 3} 3x + 9/(x^2 - 9)$ exists by graphing f(x). Where f(x)=3x+9/(x^2-9)

X	f(x)

x	f(x)

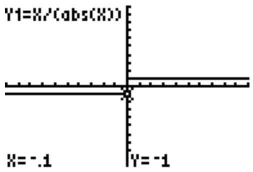
5) Decide if the limits exist:
(For the radical press ψ and ϒ)

a) $\lim_{x\to 1}\frac{\sqrt{5-x}-3}{\sqrt{10-x}-3}$ **b)** $\lim_{x\to 0} x\sin(\frac{1}{x})$ **c)** $\lim_{x\to 0}\frac{\tan x}{\sin x}$

One sided limits exist but they are not equal

6)

Lets consider f(x) = x/|x|. Graph this function. For the absolute value press and choose num and choose abs (or just pres 1). Your graph should look like this. Enter positive and negative values for x.



Write down the values(if there exists) for one sided limits.

$\lim_{x\to 0+} f(x)$=
$\lim_{x\to 0-} f(x)$=

Do the one sided limits exist?

Are they equal to eachother?

Does the limit exist?

If not why not?

Real life application

Assume you are giving private lessons and you charge 20 TL for each hour with maximum charge of 120 TL for half of the day(discounts mean more clients, be smart!). If f(t) equals your total charge for t hours sketch a graph of y=f(t) for 0<=t<=12. Decide the limits $\lim_{t \to 2.5} f(t)$ and $\lim_{t \to 3} f(t)$ exist and if they do determine the values.

<u>Reflection</u>

I was going to do both limits and continuity but the worksheet would be crowded and overwhelming fort he students.

I chose to do limits because it is the core of calculus. We use it for derivatives, sequences, series and integrals. It is most important subject of calculus. In my worksheet students will do a lot of graphing and the fact that we don't have to use any applications is good for the students because they will not have a hard time trying to learn and use new applications.

In this worksheet, the calculator satisfies almost all of the roles of graphing calculators which are: transformational tool, computational tool, visualization tool, data collection and analysis tool and discovery tool. Calculator does most of the work so the students have more time for understanding the concept. The calculator helps to compute evaluate the limits. It is dynamic and has visual capabilities. It is able to collect real time data and students can discover new things with the help of the calculator.